Lesson 26: Recursive Challenge—The Double and Add-5 Game

The “double and add 5” game is *loosely* related to the Collatz conjecture—an *unsolved* conjecture in mathematics named after Lothar Collatz, who first proposed the problem in 1937. The conjecture includes a recurrence relation, “triple and add 1,” as part of the problem statement. A worthwhile activity for you and your class is to read about the conjecture online.

Classwork

**Example 1**

Fill in the “doubling and adding 5” below:

|  |  |
| --- | --- |
| Number | Double and add 5 |
|  |  |
|  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

Exercise 1

Complete the tables below for the given starting number.

|  |  |
| --- | --- |
| Number | Double and add 5 |
|  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

|  |  |
| --- | --- |
| Number | Double and add 5 |
|  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

Exercise 2

Given a starting number, double it and add 5 to get the result of round 1. Double the result of Round 1 and add 5, and so on. The goal of the game is to find the smallest starting whole number that produces a result of 100 or greater in 3 rounds or less.

Exercise 3

Using a generic initial value, , and the recurrence relation, , for , find a formula for in terms of .

Problem Set

1. Write down the first 5 terms of the recursive sequences defined by the initial values and recurrence relations below:  
   1. and , for ,
   2. and , for ,
   3. and , for ,
   4. and , for ,
   5. and , for ,
   6. and , for ,
   7. and , for ,
   8. and , for ,
   9. and , for ,
2. Look at the sequences you created in problems 1b through 1d. How would you define a recursive sequence that generates multiples of 31?
3. Look at the sequences you created in problems 1e through 1g. How would you define a recursive sequence that generates powers of 15?
4. The following recursive sequence was generated starting with an initial value of , and the recurrence relation , for . Fill in the blanks of the sequence:

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1. For the recursive sequence generated by initial value, , and recurrence relation, , for , find a formula for in terms of . Describe in words what this sequence is generating.
2. For the recursive sequence generated by initial value, , and recurrence relation, , for , find a formula for in terms of .